LETTERS TO THE EDITOR.

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Corrections of Maximum and Ex-Meridian Altitudes.

IN navigation the error introduced by taking the maximum altitude of a heavenly body for its meridian altitude is not sufficiently great to need correction when it is due to variation of declination alone, as it is then much within the probable errors of observation. When, however, a ship is steaming at a high speed, the error is considerably increased by the variation of latitude, especially when this is of opposite sign to the variation of declination.

The formula giving the correction in seconds of arc to be applied to the zenith distance of a body for reduction to the meridian is

$$x = \mathrm{C}h^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where, with the usual notation $\mathrm{C} = \dfrac{\cos l \cos \delta}{2 \sin (l \mp \delta)} \cdot \dfrac{\sin^2 15'}{\sin 1''}$, and h is the hour angle in minutes of time. Thus, if z be the zenith distance, we have the equation

$$z - \mathrm{C}h^2 = l \mp \delta,$$

the upper or lower sign being taken according as l and δ are of the same or of opposite sign. Since we may consider z, h, l, and δ as functions of the mean time t and C constant, we have on differentiation, if z be a minimum,

$$-2\mathrm{C}h\frac{dh}{dt} = \frac{dl}{dt} \mp \frac{d\delta}{dt}.$$

Let H and H′ be the hour angles of the body and the ship's zenith measured from some fixed meridian, and let u, v denote the northerly and westerly components of the ship's velocity in knots; then $\dfrac{dh}{dt} = \dfrac{d\mathrm{H}}{dt} - \dfrac{d\mathrm{H}'}{dt}$ and $\dfrac{dl}{dt} = v$. Also if the body's projection on the Earth's equator moves round at the rate of U knots,

$$\frac{d\mathrm{H}'}{dt}\Big/\frac{d\mathrm{H}}{dt} = u \sec l/\mathrm{U}\ ;$$

and the above equation becomes

$$-2\mathrm{C}h\frac{d\mathrm{H}}{dt}(1 - u \sec l/\mathrm{U}) = v \mp \dot{\delta} \quad . \quad . \quad . \quad (2)$$

Now this equation will assume different forms according as the body under consideration is the Sun, a star, or the Moon. For, since the motion of the ship is expressed in mean solar time, the motion of the body must likewise be expressed in that time.

The unit of time being a minute of mean time, and the unit of arc a second of arc, we have for the Sun

$$\frac{d\mathrm{H}}{dt} = 1 \text{ and } \mathrm{U} = 900,$$

so that (2) becomes

$$-2\mathrm{C}h(1 - u \sec l/900) = v \mp \dot{\delta}.$$

For latitude 60° and $u = 20$ the value of $u \sec l/900$ is about ·04, so that we see for ordinary speeds and ordinary latitudes reached the term involving u may be neglected, and the equation reduces to

$$-2\mathrm{C}h = v \mp \dot{\delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For a star

$$\frac{d\mathrm{H}}{dt} = \frac{\text{length of mean solar day}}{\text{length of sidereal day}} = 1$$

with sufficient accuracy, and as before we obtain equation (3).

For the Moon

$$\frac{d\mathrm{H}}{dt} = \frac{\text{length of mean solar day}}{\text{length of lunar day}} = \frac{1}{1{\cdot}035}$$

and

$$\mathrm{U} = 869,$$

so that in this case (2) becomes

$$-2\mathrm{C}h = 1{\cdot}035\,(v \mp \dot{\delta}).$$

On giving C its proper value, and putting w for the velocity compounded of the velocity in latitude of the ship and the velocity in declination of the body (3) becomes

$$h = -w(\tan l \mp \tan \delta) \sin 1'' \operatorname{cosec}^2 15' \quad . \quad . \quad (5)$$

The reduction is therefore from (1)

$$x = -\frac{w^2}{2}(\tan l \mp \tan \delta) \sin 1'' \operatorname{cosec}^2 15' \quad . \quad . \quad (6)$$

Equations (5) and (6) therefore give the hour angle (from the ship's meridian) and reduction for the Sun or a star at the maximum altitude. For the Moon $1{\cdot}035w$ must be substituted for w. The form of these equations has suggested the construction of the accompanying table, which gives the value of

°		°		°	
1	·004	21	·098	41	·221
2	·009	22	·103	42	·229
3	·013	23	·108	43	·237
4	·017	24	·113	44	·245
5	·022	25	·118	45	·254
6	·026	26	·124	46	·263
7	·031	27	·129	47	·273
8	·035	28	·135	48	·283
9	·040	29	·141	49	·293
10	·045	30	·147	50	·303
11	·049	31	·153	51	·314
12	·054	32	·159	52	·326
13	·059	33	·165	53	·338
14	·063	34	·171	54	·350
15	·068	35	·178	55	·363
16	·073	36	·185	56	·377
17	·078	37	·192	57	·392
18	·083	38	·199	58	·407
19	·088	39	·206	59	·424
20	·093	40	·213	60	·441

$\tan x^\circ \sin 1'' \operatorname{cosec}^2 15'$ as far as $x = 60$. Thus in latitude l° when a body (Sun or star) of declination δ° is at its maximum altitude, the sum or difference (according as l and δ are of different or of the same name) of the arguments corresponding to l and δ multiplied by w gives the hour-angle in minutes of time: the sum or difference multiplied by $\dfrac{w^2}{2}$ gives the reduction in seconds of arc. For the hour angle of the Moon the sum or difference must be multiplied by $1{\cdot}035w$, and for the reduction by $\frac{1}{2}(1{\cdot}035w)^2$.

Example.—D.R. latitude 48° N. Moon's declination 18° 48′ S. decreasing 120″ per 10m., ship steaming S. 20° E. 16 knots.

Remembering that when the ship is in N. latitude, and steaming towards south

$$w = -v \mp \dot{\delta},$$

we have

$$w = -27,$$

and

$$h = 27 \times 1{\cdot}035({\cdot}283 + {\cdot}087) = 10{\cdot}3\text{m}$$
$$x = 144''$$

If a ship whose maximum speed is 20 knots does not reach a higher latitude than 60, the greatest values that h and x can have, are for the Sun, 9m. 15s. and 1′ 37″; for the Moon, 17m. 20s. and 5′ 41″.

I will now investigate the nature of a diagram[1] from which the reduction in ordinary ex-meridian observations may be obtained, as well as the hour angle and reduction for maximum altitudes. It is found convenient for this purpose to express x in minutes of arc, when (1) becomes

$$h^2 = 2x(\tan l \mp \tan \delta) \sin 1' \operatorname{cosec}^2 15' \quad . \quad . \quad . \quad (7)$$

Now if a circle of radius a be referred to a point in its circumference as origin, its equation is

$$r^2 = 2ax,$$

r being the radius vector and x the abscissa measured along the diameter through the origin. Comparing this with (7) we see that if a system of circles be described passing through a common origin O, their centres being collinear and on the same side of O, the reduction x is the abscissa of the point whose radius vector is h on one of these circles: the particular circle on which the point lies being that whose radius is the sum or difference of the ordinates of the curve

$$y = \tan x \sin 1' \operatorname{cosec}^2 15'$$

corresponding to

$$x = l \text{ and } x = d.$$

1 A diagram based on the properties of the parabola was given by Prof. Foscolo, of Venice, and published by the Hydrographic Office about 1870.

It will be noticed that when

$$h > 2(\tan l \pm \tan \delta) \sin 1' \operatorname{cosec}^2 15',$$

the above construction fails, and x may be then considered as twice the abscissa of the point on the circle whose radius is

$$2(\tan l \mp \tan \delta) \sin 1' \operatorname{cosec}^2 15',$$

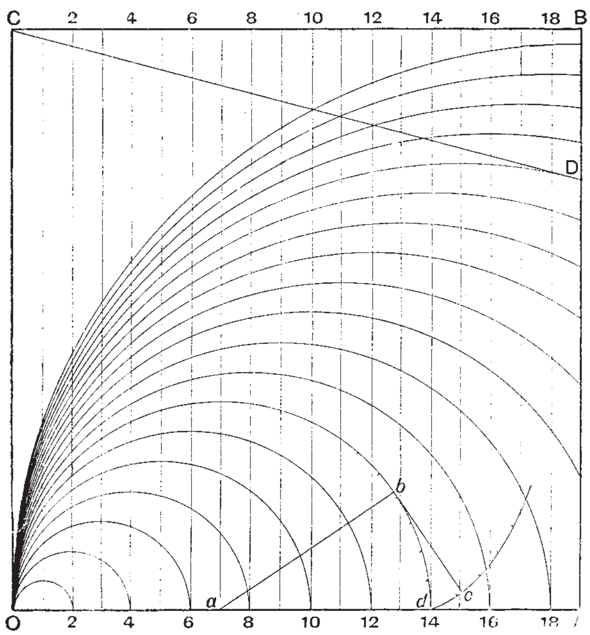
By making a diagram on ruled paper, the distance between the lines being taken as unit, the difficulty of measuring x and h is more or less overcome.

A portion of such a diagram is represented in the annexed figure, C D being a portion of the curve

$$y = \tan x \sin 1' \operatorname{cosec}^2 15'$$

described with C as origin, y being for convenience reckoned negatively.

As an example of its use, suppose it were required to find the reduction for latitude 17° N., declination 9° S., and hour-angle 12m. By using a pair of dividers the sum of the ordinates of the curve C D corresponding to 9 and 17 is found to be



7, and a radius vector of length 12 meets the circle of radius 7 at a point whose abscissa is 10. The required reduction is therefore 10'.

Again, x being now expressed in minutes of arc (5) and (6) may be written

$$h = -\frac{w}{60}(\tan l \mp \tan \delta) \sin 1' \operatorname{cosec}^2 15'$$

$$x = \left(\frac{w}{60}\right)^2(\tan l \mp \tan \delta) \sin 1' \operatorname{cosec}^2 15'.$$

So that h and x are the lengths of the arcs of a circle and its involute respectively, corresponding to the angle at the centre whose circular measure is $w/60$ in the case of the Sun, and $1{\cdot}035w/60$ in the case of the Moon. Now for the former body w will not in general exceed 20, and for the latter 38, so that we may assume for graphical purposes that in the case of the Sun w is the number of degrees in an angle of circular measure $w/60$, and in the case of the Moon w is the number of degrees in an angle of circular measure $1{\cdot}035w/60$.

Thus in both cases h is approximately the arc of a circle of radius

$$(\tan l \mp \tan \delta) \sin 1' \operatorname{cosec}^2 15'$$

intercepted between the diameter and the radius which makes an angle w degrees with it.

Having obtained h, x may be found as in the general case, or by drawing a tangent to meet the involute, as in the figure.

In the preceding example, suppose the ship to be steaming south 20 knots, the declination decreasing 160″ per 10m. Here $w = -36$. By laying off at an angle of 36°, the intercepted arc bd is found to be about 4·3, and by drawing the tangent at b the intercepted arc of the involute is about 1·3. The hour angle and reduction for the maximum altitude are therefore 4m.·3 and 1′·3 respectively.

The graphical method considered above will be found to give results which, although approximate, are sufficiently accurate for purposes of navigation; in fact, if the diagram be constructed on a large scale, the reduction may be easily obtained within fifteen seconds of the truth.

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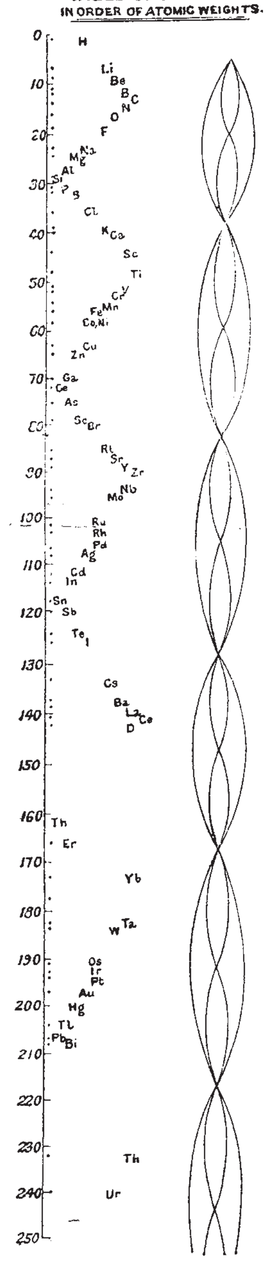
Argon and the Periodic System.

THE annexed engraving is a copy, on a small scale, of a large diagram which I have used with advantage for some years in dealing with the periodic classification of the elements. It may prove of some little interest to your readers who are actively discussing the probable position of "argon," on the assumption that this remarkable substance is an element.

TABLE OF ELEMENTS.
IN ORDER OF ATOMIC WEIGHTS.



To the left of the illustration is a scale of equal parts, and the dots indicate the atomic weights of the elements the symbols of which are placed farther to the right. The latter are arranged in zig-zag fashion so as to exhibit the periodic rise and fall in general properties, observed in each set of seven elements. A certain analogy may be traced between these periods and the loops into which a suspended cord of somewhat unequal weight can be thrown when set in vibration. Each small loop pictures for us a small period; and, just as the *alternate* loops are those which are in the same phase at any given moment, so the alternate periods of the elements are those between which the closest resemblances can be traced.

The members of Mendeléef's eighth group, or the "triplets," as they are sometimes called, viz. Fe, Ni, and Co, with atomic weights from 56 to 59: Ru, Rh and Pd, 102 to 106: Os, Ir and Pt, 191 to 195, seem to form another system of elements which—to pursue the analogy of the vibrating cord—is related to that of the other elements somewhat as a given note to its octave. On carrying the eye along the curves it will be seen that the atomic weights of triplets occur nearly opposite to the points of maximum displacement of three of the greater loops. We know very little as yet about the elements the atomic weights of which lie between 140 and 180, hence we cannot recognise the triplets the atomic weights of which should be near to 150; and a similar remark applies to the elements above 210. But the distribution of the triplets throughout the whole of the best-known elements is so nearly regular that it is difficult to avoid the inference that *three* elements should also be found in the symmetrical position between 19 and 23, *i.e.* between fluorine and sodium. And further, that